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THREE-DIMENSIONAL BOUNDARY LAYER IN A PARTLY IONIZED MULTICOMPONENT GAS

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A system of equations is derived for the three-dimensional boundary layer in a partly ionized multicomponent gas with frozen reactions under conditions of quasi-inertness and absence of external electromagnetic fields and of energy transfer by radiation. An analytical computation method based on the use of successive approximations is investigated. Variation of transfer coefficients across the boundary layer is taken into account by approximating the values of these at the external boundary and at the surface of the body. First approximation values of surface friction and heat exchange coefficients are obtained for the locally self-similar cases. An example of computation of the flow of frozen air past a cone with spherically blunted nose at an angle of attack is presented.

1. Let us consider the three-dimensional motion of a partly ionized multicomponent gas. If external electromagnetic fields are absent and the thermal diffusion effect is disregarded, the system of equations for a three-dimensional frozen boundary layer can be written as follows:

$$\frac{\partial}{\partial \xi}\left(\rho \sqrt{\frac{g}{g_{11}}}\, u\right) + \frac{\partial}{\partial \eta}\left(\rho \sqrt{\frac{g}{g_{22}}}\, w\right) + \sqrt{g}\,\frac{\partial \rho v}{\partial \zeta} = 0$$

$$\frac{\rho u}{\sqrt{g_{11}}}\frac{\partial c_i}{\partial \xi} + \frac{\rho w}{\sqrt{g_{22}}}\frac{\partial c_i}{\partial \eta} + \rho v \frac{\partial c_i}{\partial \zeta} + \frac{\partial I_i}{\partial \zeta} = 0, \quad i = 1, \ldots, N$$

$$\frac{u}{\sqrt{g_{11}}}\frac{\partial u}{\partial \xi} + \frac{w}{\sqrt{g_{22}}}\frac{\partial u}{\partial \eta} + v\frac{\partial u}{\partial \zeta} + A_1 u^2 + A_2 w^2 + A_3 uw =$$
$$\frac{A_4}{\rho} + \frac{1}{\rho}\frac{\partial}{\partial \zeta}\left(\mu \frac{\partial u}{\partial \zeta}\right), \quad \frac{\partial p}{\partial \zeta} = 0$$

$$\frac{u}{\sqrt{g_{11}}}\frac{\partial w}{\partial \xi} + \frac{w}{\sqrt{g_{22}}}\frac{\partial w}{\partial \eta} + v\frac{\partial w}{\partial \zeta} + B_1 u^2 + B_2 w^2 + B_3 uw =$$
$$\frac{B_4}{\rho} + \frac{1}{\rho}\frac{\partial}{\partial \zeta}\left(\mu \frac{\partial w}{\partial \zeta}\right)$$

$$\frac{\rho u}{\sqrt{g_{11}}}\frac{\partial H}{\partial \xi} + \frac{\rho w}{\sqrt{g_{22}}}\frac{\partial H}{\partial \eta} + \rho v \frac{\partial H}{\partial \zeta} =$$

$$\frac{\partial}{\partial\zeta}\left\{\frac{\mu}{\sigma}\left[\frac{\partial H}{\partial\zeta}+(\sigma-1)\frac{\partial}{\partial\zeta}\frac{U^2}{2}-\sum_{k=1}^{N}h_k\left(\frac{\sigma}{\mu}I_k+\frac{\partial c_k}{\partial\zeta}\right)\right]\right\}$$

$$h_i=\int_0^T c_{pi}\,dT+h_i^\circ,\quad H=\sum_{k=1}^{N}c_kh_k+\frac{U^2}{2}$$

where the first equation is that of continuity, the second is the equation of diffusion, the third, fourth and fifth are equations of motion of the mixture, and the sixth is the equation of energy. System (1.1) is closed by the Stefan-Maxwell relationships and the equation of state of the gas mixture

$$\rho\frac{\partial x_i}{\partial\zeta}=m\sum_{k=1}^{N}\frac{I_k}{m_k}\left[\frac{x_i}{D_{ik}}-\delta_{ik}\sum_{l=1}^{N}\frac{x_l}{D_{kl}}+\frac{x_ie_i}{L}\sum_{j=1}^{N}\frac{x_j}{D_{kj}}(e_k-e_j)\right],\qquad(1.2)$$

$$L=\sum_{k=1}^{N}x_ke_k^2,\quad i=1,\ldots,N$$

$$p=\frac{\rho RT}{m},\quad m=\sum_{i=1}^{N}x_im_i\qquad(1.3)$$

where ξ, η, ζ are coordinates of an orthogonal system, with the ζ-axis normal to the body surface so that the surface $\zeta=0$ coincides with the surface of the body, and the ξ- and η-axes directed along the body surface; g_{11} and g_{22} are components of the metric tensor $g=g_{11}g_{22}$; N is the number of components in the mixture; u, v and w are components of the mean mass flow rate $\mathbf{V}$ along the ξ-, η- and ζ-axes, respectively; p, ρ and T are, respectively, the pressure, density and absolute temperature of the mixture; m is the molecular weight of the mixture; c_i, x_i, m_i and e_i are the mass and molecular concentration, the molecular weight and the electrical charge of the i-th component; I_i is the projection of the mass diffusion stream of the i-th component on the ζ-axis; μ and σ are the viscosity coefficient and the Prandtl number, respectively; D_{ij} are the binary diffusion coefficients; h_i is the enthalpy of the i-th component; c_{pi} is the specific heat at constant pressure of the i-th component; h_i° is the specific heat of formation of the i-th component, and H is the over-all enthalpy of the mixture.

Coefficients A_i and B_i $(i=1,\ldots,4)$ are determined by the body geometry and the external flow [1].

The system of Eqs. (1.1) — (1.3) must be supplemented by the identities

$$\sum_{k=1}^{N}x_k=\sum_{k=1}^{N}c_k=1,\quad \sum_{k=1}^{N}I_k=0\qquad(1.4)$$

The system of Eqs. (1.1) — (1.3) with (1.4) is a closed system of $2N+6$ independent equations with p, ρ, u, w, v, H, $c_1,\ldots,c_N$ and $I_1,\ldots,I_N$ as the unknowns.

Boundary conditions at the external boundary of the boundary layer and at the wall are

$$u\to u_e(\xi,\eta),\quad w\to w_e(\xi,\eta),\quad T\to T_e(\xi,\eta)\qquad(1.5)$$

$$c_i(\xi,\eta)\to c_{ie}=\text{const}\quad \text{for } \zeta\to\infty$$

$$u=w=0,\quad T=T_w(\xi,\eta)\quad\text{(at the wall)}$$

If the wall is impermeable, then for the diffusion streams of mixture elements

$$I_j^* \to 0 \quad \text{for} \quad \zeta \to 0 \tag{1.6}$$

We assume that the gas at the wall is in a state of chemical equilibrium.

In this case the system of equations that determine the equilibrium composition, the Stefan-Maxwell formulas, and the conditions (1.6) of wall impermeability provide sufficient conditions for determining the mixture chemical composition c_i^w and of the diffusion streams of components.

The Stefan-Maxwell formulas (1.2) take into account the electric fields induced by charge separation. Its intensity is determined by the condition of quasi-neutrality of the gas.

Having solved the problem it becomes possible to determine the viscous friction stress distribution at the surface of the body and the total convection heat flux to the wall

$$\tau_{11} = \mu \frac{\partial u}{\partial \zeta}\bigg|_{\zeta=0}, \quad \tau_{22} = \mu \frac{\partial w}{\partial \zeta}\bigg|_{\zeta=0} \tag{1.7}$$

$$-I_{qw} = \left(\lambda \frac{\partial T}{\partial \zeta} + \sum_{k=1}^{N_r} Q_k I_k\right)_{\zeta=0} \tag{1.8}$$

2. We substitute the self-similar coordinate λ for ζ, and pass to dimensionless functions

$$\lambda = \sqrt{\frac{u_e}{\mu_e \rho_e \alpha}} \int_0^\zeta \rho \, d\zeta \tag{2.1}$$

$$u = u_e(\xi, \eta) E(\xi, \eta, \lambda), \quad w = \beta(\xi, \eta) u_e(\xi, \eta)(G + \varphi E)$$

$$\rho v = \sqrt{\frac{\mu_e \rho_e u_e}{\alpha}} \left[K - \frac{\alpha}{\sqrt{g_{11}}} E \frac{\partial \lambda}{\partial \xi} - \frac{\alpha\beta}{\sqrt{g_{22}}} (G + \varphi E) \frac{\partial \lambda}{\partial \eta}\right]$$

$$H = H_w + (H_e - H_w)\theta, \quad \varphi = \frac{w_e}{\beta u_e}$$

$$c_k = c_k^w + (c_k^e - c_k^w) z_k, \quad I_i = \sqrt{\frac{u_e \rho_e \mu_e}{\alpha}} (c_i^e - c_i^w) X_i$$

where $\alpha = \alpha\ (\xi, \eta)$ and $\beta = \beta\ (\xi, \eta)$ are, so far, arbitrary functions. After substitution and related transformations system (1.1)—(1.3) assumes the form

$$\frac{\partial K}{\partial \lambda} = -P_1^* E - P_2^* G - N_4 \frac{\partial E}{\partial \xi} - N_5 \frac{\partial G}{\partial \eta} - \varphi N_5 \frac{\partial E}{\partial \eta} \tag{2.2}$$

$$\frac{\partial}{\partial \lambda}\left(l \frac{\partial E}{\partial \lambda}\right) = K \frac{\partial E}{\partial \lambda} + N_1^* \left(E^2 - \frac{\rho_e}{\rho}\right) +$$

$$N_2^* G^2 + N_3^* EG + N_4 E \frac{\partial E}{\partial \xi} + N_5 (G + \varphi E) \frac{\partial E}{\partial \eta}$$

$$\frac{\partial}{\partial \lambda}\left(l \frac{\partial G}{\partial \lambda}\right) = K \frac{\partial G}{\partial \lambda} + M_1^* \left(E^2 - \frac{\rho_e}{\rho}\right) +$$

$$M_2^* G^2 + M_3^* EG + N_4 E \frac{\partial G}{\partial \xi} + N_5 (G + \varphi E) \frac{\partial G}{\partial \eta}$$

$$\frac{\partial}{\partial \lambda}\left(\frac{l}{\sigma} \frac{\partial \theta}{\partial \lambda}\right) = K \frac{\partial \theta}{\partial \lambda} + \frac{\partial}{\partial \lambda}\left\{\frac{1-\sigma}{k(1-t_0)} \frac{l}{\sigma} \frac{\partial}{\partial \lambda}[E^2 + \beta^2 (G + \varphi E)^2] + \right.$$

$$\sum_{i=1}^{N} h_i \frac{c_i^e - c_i^w}{H_e - H_w}\left(X_i + \frac{l}{\sigma}\frac{\partial z_i}{\partial \lambda}\right)\Big\} +$$

$$\frac{1-\theta}{1-t_0} E\left(N_4 \frac{\partial t_0}{\partial \xi} + \varphi N_5 \frac{\partial t_0}{\partial \eta}\right) + \frac{1-\theta}{1-t_0} G N_5 \frac{\partial t_0}{\partial \eta} +$$

$$N_4 E \frac{\partial \theta}{\partial \xi} + N_5 (G + \varphi E)\frac{\partial \theta}{\partial \eta},$$

$$-\frac{\partial X_i}{\partial \lambda} = K \frac{\partial z_i}{\partial \lambda} + Q_{1i}^*(1 - z_i)E +$$

$$Q_{2i}^* G(1 - z_i) + N_4 E \frac{\partial z_i}{\partial \xi} + N_5 (G + \varphi E)\frac{\partial z_i}{\partial \eta}, \qquad i = 1, \ldots, N$$

$$\frac{\partial z_i}{\partial \lambda} = \sum_{j=1}^{N} \frac{X_j m}{l}\left\{\frac{c_j}{m_j}\sum_{k=1}^{N}\left(\frac{m_j}{m_k} - 1\right) c_k Sh_{jk} - \delta_{jk}\sum_{k=1}^{N}\frac{c_k}{m_k} Sh_{ik} +\right.$$

$$\left.\frac{c_i(e_i - L_1)}{L_2}\sum_{q=1}^{N}\frac{c_q}{m_q} Sh_{jq}(e_j - e_q)\right\}$$

$$Q_{1i}^* = \frac{\alpha}{\sqrt{g_{11}}}\frac{1}{c_i^e - c_i^w}\frac{\partial c_i^w}{\partial \xi} + \frac{\varphi\alpha\beta}{\sqrt{g_{22}}}\frac{1}{c_i^e - c_i^w}\frac{\partial c_i^w}{\partial \eta}$$

$$Q_{2i}^* = \frac{\alpha\beta}{\sqrt{g_{22}}}\frac{1}{c_i^e - c_i^w}\frac{\partial c_i^w}{\partial \eta}$$

$$l = \frac{\mu\rho}{\mu_e\rho_e}, \quad t_0 = \frac{H_w}{H_e}, \quad k = \frac{2H_e}{u_e^2}, \quad \mathrm{Sh}_{ij} = \frac{\mu}{\rho D_{ij}}$$

$$L_1 = \sum_{i=1}^{N} c_i e_i, \quad L_2 = \sum_{i=1}^{N}\frac{c_i e_i^2}{m_i}$$

The coefficients $N_1^*, N_2^*, N_3^*, N_4, N_5, M_1^*, M_2^*, M_3^*, P_1^*$ and P_2^* are of the same form as in the case of compressible boundary layer in a one-component gas [1]. They depend only on parameters of the external flow and the geometry of the body.

The boundary conditions in dimensionless variables are

$$E = G = K = \theta = z_i = 0 \qquad \text{for } \lambda = 0 \tag{2.3}$$

$$E \to 1,\ G \to 0,\ \theta \to 1,\ z_i \to 1 \qquad \text{for } \lambda \to \infty \tag{2.4}$$

3. We integrate the equations of motion, energy and diffusion of system (2.2) with respect to the λ-coordinate from some of its value to infinity, taking into account boundary conditions (2.4). The system of equations now becomes

$$-l\frac{\partial E}{\partial \lambda} = -K(E - 1) + (P_1^* + N_1^*)\theta_{11} + N_1^*\theta_1 + \tag{3.1}$$

$$(P_2^* + N_3^*)\theta_{21} - P_2^*\theta_2 + N_1^*\theta_\rho + N_2^*\theta_{22} + N_4 \frac{\partial \theta_{11}}{\partial \xi} +$$

$$N_5 \frac{\partial \theta_{12}}{\partial \eta} + \varphi N_5 \frac{\partial \theta_{11}}{\partial \eta}$$

$$-l\frac{\partial G}{\partial \lambda} = -KG + M_1^*(\theta_{11} + \theta_1 + \theta_\rho) + (P_2^* + M_2^*)\theta_{22} + \tag{3.2}$$

$$(P_1{}^* + M_3{}^*)\theta_{21} + N_4 \frac{\partial\theta_{21}}{\partial\xi} + N_5 \frac{\partial\theta_{22}}{\partial\eta} + \varphi N_5 \frac{\partial\theta_{21}}{\partial\eta}$$

$$-\frac{l}{\sigma}\frac{\partial\theta}{\partial\lambda} = -K(\theta - 1) + (P_1{}^* - R_1{}^*)\theta_{31} + (P_2{}^* - R_2{}^*)\theta_{32} + \qquad (3.3)$$

$$N_4 \frac{\partial\theta_{31}}{\partial\xi} + N_5 \frac{\partial\theta_{32}}{\partial\eta} + \varphi N_5 \frac{\partial\theta_{31}}{\partial\eta} -$$

$$\frac{1-\sigma}{k(1-t_0)}\frac{l}{\sigma}\frac{\partial}{\partial\lambda}[E^2 + \beta^2 (G + \varphi E)^2] -$$

$$\sum_{i=1}^{N} h_i \frac{c_i^{\,e} - c_i^{\,w}}{H_e - H_w}\left(X_i + \frac{l}{\sigma}\frac{\partial z_i}{\partial\lambda}\right)$$

$$X_i = -K(z_i - 1) + \qquad (3.4)$$

$$(P_1{}^* - Q_{1i}{}^*)S_{1i} + (P_2{}^* - Q_{2i}{}^*)S_{2i} + N_4 \frac{\partial S_{1i}}{\partial\xi} +$$

$$N_5 \frac{\partial S_{2i}}{\partial\eta} + \varphi N_5 \frac{\partial S_{1i}}{\partial\eta}$$

$$R_1{}^* = \frac{N_4}{1-t_0}\frac{\partial t_0}{\partial\xi} + \frac{\varphi N_5}{1-t_0}\frac{\partial t_0}{\partial\eta}, \quad R_2{}^* = \frac{N_5}{1-t_0}\frac{\partial t_0}{\partial\eta}$$

$$S_{1i} = \int_\lambda^\infty (z_i - 1)E\,d\lambda, \quad S_{2i} = \int_\lambda^\infty (z_i - 1)G\,d\lambda, \quad \theta_\rho = \int_\lambda^\infty \left(1 - \frac{\rho_e}{\rho}\right)d\lambda$$

Integrals $\theta_1, \theta_2, \theta_{11}, \theta_{12}, \theta_{21}, \theta_{22}, \theta_{31}$ and θ_{32} are of the same form as for a compressible homogeneous gas [2]. We eliminate streams X_i from the equations by substituting expressions for dimensionless streams (3.4) into the energy equation (3.3) and into the Stefan-Maxwell formulas (2.2). We integrate the obtained system of equations with respect to λ from zero to some value of λ taking into account (2.3), and obtain a system of integro-differential equations whose solution with boundary conditions (2.4) is equivalent to the solution of system (2.2) with conditions (2.3) and (2.4). From the continuity equation we obtain the expression for K which we substitute into the remaining equations and, thus, eliminate it from the system. We solve the derived system of equations by the method of successive approximations as was done in [2] for a compressible homogeneous fluid.

Let us assume that the n-th approximation is known. Substituting it into the equations of the system and carrying out the appropriate integration, we obtain the $(n+1)$-st approximation. To have the boundary conditions satisfied by the obtained $(n+1)$-st approximation we introduce controlling functions $\delta^{(n)}(\xi, \eta)$, $b^{(n)}(\xi, \eta)$, $d^{(n)}(\xi, \eta)$ and $y_i^{(n)}(\xi, \eta)$ We have

$$\zeta = \lambda / \sqrt{\delta}, \quad E = E(\xi, \eta, \zeta), \quad G = bG(\xi, \eta, \zeta) \qquad (3.5)$$

$$\theta - E = d\,[\theta(\xi, \eta, \zeta) - E(\xi, \eta, \zeta)]$$

$$z_i - E = y_i\,[z_i(\xi, \eta, \zeta) - E(\xi, \eta, \zeta)], \; i = 1, \ldots, N$$

Here and below the superscript (n) is omitted for brevity. The system of equations for the determination of the $(n+1)$-st approximation is of the form

$$-E^{(n+1)} = \delta\,[A_1 + bB_1 + b^2 C_1] + a_{11}\frac{\partial\delta}{\partial\xi} + a_{12}\frac{\partial\delta}{\partial\eta} + \qquad (3.6)$$

$$a_{14}\frac{\partial b}{\partial \eta}+\sqrt{\delta}A_{01}, \quad -G^{(n+1)}=\delta\left[A_2+bB_2+b^2C_2\right]+$$

$$a_{21}\frac{\partial \delta}{\partial \xi}+a_{22}\frac{\partial \delta}{\partial \eta}+a_{23}\frac{\partial b}{\partial \xi}+a_{24}\frac{\partial b}{\partial \eta}+b\sqrt{\delta}A_{02}$$

$$z_i^{(n+1)}=\delta\left[A_{3+i}+bB_{3+i}+\sum_{j=1}^{N}y_jC_{3+i,j}+b\sum_{j=1}^{N}y_jD_{3+i,j}\right]+$$

$$a_{3+i,1}\frac{\partial \delta}{\partial \xi}+a_{3+i,2}\frac{\partial \delta}{\partial \eta}+a_{3+i,4}\frac{\partial b}{\partial \eta}+\sum_{j=1}^{N}a_{3+i,5,j}\frac{\partial y_j}{\partial \xi}+$$

$$\sum_{j=1}^{N}a_{3+i,6,j}\frac{\partial y_j}{\partial \eta}+\sqrt{\delta}A_{0,3+i}, \quad i=1,\ldots,N$$

$$\theta^{(n+1)}=\delta\left[A_3+bB_3+dC_3+bdD_3+\sum_{i=1}^{N}E_{3i}y_i+b\sum_{i=1}^{N}G_{3i}y_i\right]+$$

$$\sum_{i=1}^{N}F_{3i}y_i+T_3+a_{31}\frac{\partial \delta}{\partial \xi}+a_{32}\frac{\partial \delta}{\partial \eta}+a_{34}\frac{\partial b}{\partial \eta}+a_{35}\frac{\partial d}{\partial \xi}+$$

$$a_{36}\frac{\partial d}{\partial \eta}+\sum_{i=1}^{N}a_{37,i}\frac{\partial y_i}{\partial \xi}+\sum_{i=1}^{N}a_{38,i}\frac{\partial y_i}{\partial \eta}+\sqrt{\delta}A_{03}$$

where the coefficients A_1, B_1, . . . are double integrals.

From equations of system (3.6) with allowance for (2.4) we obtain for $\zeta\to\infty$ a system of equations in partial derivatives with respect to δ, b, d and y_i. The first approximation coefficients $A_{1\infty}$, $B_{1\infty}$, . . derived from coefficients A_1, B_1, . . . at $\zeta\to\infty$, are defined below for the locally self-similar case. The substitution of the calculated values of controlling functions into Eqs. (3.6) shows that the $(n+1)$-st approximation satisfies boundary conditions (2.4). This process is repeated until the specified convergence of the approximation sequence is reached.

4. Let us consider the locally self-similar case [2, 3]. For this we introduce effective ambipolar diffusion coefficients [4] and represent the Stefan-Maxwell formulas (1.2) in the form

$$I_k=-\rho D_k^*\frac{\partial c_k}{\partial \zeta} \qquad (4.1)$$

The use of formulas (4.1) instead of (1.2) simplifies the equations of diffusion and energy.

In the considered case the system of equations in the locally self-similar approximation in the absence of blowing-in is of the form

$$-E^{(n+1)}=\delta\left[A_1+bB_1+b^2C_1\right] \qquad (4.2)$$

$$-G^{(n+1)}=\delta\left[A_2+bB_2+b^2C_2\right]$$

$$z_i^{(n+1)}=\delta\left[A_{3+i}+b\,B_{3+i}+y_iC_{3+i}+by_iD_{3+i}\right]$$

$$\theta^{(n+1)}=\delta\left[A_3+bB_3+dC_3+bdD_3\right]+\sum_{i=1}^{N}F_{3i}y_i+T_3$$

For the determination of the controlling functions we obtain a system of algebraic equations. The computation of coefficients of these equations requires the knowledge of the character of variation of parameters D_i^* across the layer.

We approximate the variation of expressions Sh_i / l, where Sh_i are the effective ambipolar Schmidt numbers, by using their values at the external boundary of the boundary layer and at the wall

$$\mathrm{Sh}_i / l = \mathrm{Sh}_i^e + (\mathrm{Sh}_i^w / l_w - \mathrm{Sh}_i^e) Z_{-1}^{\alpha_s} \quad (4.3)$$

The method of calculating the effective ambipolar diffusion coefficients at the boundaries by using the coefficients of binary diffusion is given in [4].

Here and in what follows we use functions $Z_m(\zeta)$ of the form

$$Z_{-1}(\zeta) = e^{-\zeta^2}, \quad Z_m(\zeta) = \frac{A_m}{m!} \int_{\infty}^{\zeta} (\zeta - t)^m e^{-t^2} dt, \quad m = 0, 1, \ldots$$

where A_m is determined by the condition $Z_m(0) = 1)$.

Inspection of the solution in the stagnation point neighborhood at various values of exponent α_s and its comparison with available numerical computation data at the stagnation point [5 — 7] show that it is possible to select such α_s at which the two results are in agreement. The best agreement is obtained with $\alpha_s = 1.1$.

Variation of coefficients $1 / l$, ρ_e / ρ and σ / l are defined as in (4.3). Values of transfer coefficients at the boundaries were computed in the first approximation by the Hirschfelder formulas. The required values of collision integrals for electrically neutral pairs were taken from the experimental data in [8], for charged particles from [9], and for the ion-atom pairs from [7].

The dimensionless heat flux to the wall and the coefficients of friction at the wall were determined by specifying the zero approximation as follows:

$$E^{(0)} = 1 - Z_0(\zeta), \quad G^{(0)} = b^{(0)}(\xi, \eta)[Z_0(\zeta) - Z_{-1}(\zeta)]$$

$$\theta^{(0)} = 1 - Z_0(\zeta) + d^{(0)}[Z_0(\zeta) - Z_{-1}(\zeta)]$$

$$z_i^{(0)} = 1 - Z_0(\zeta) + y_i^{(0)}[Z_0(\zeta) - Z_{-1}(\zeta)]$$

Coefficients of the system for determining the controlling functions in zero approximation are

$$A_{1\infty}^{(0)} = -0.25 P_1^* + 0.0453 N_1^* + \Delta_1(0.031 N_1^* - 0.161 P_1^*) - (0.455 + 0.357 \Delta_1) N_1^* \rho_e / \rho_w$$

$$B_{1\infty}^{(0)} = 0.104 P_2^* - 0.194 N_2^* + \Delta_1 (0.0721 P_2^* - 0.134 N_2^*)$$

$$C_{1\infty}^{(0)} = N_2^* (0.048 + 0.0382 \Delta_1)$$

$$A_{2\infty}^{(0)} = M_1^* [0.0454 + 0.0311 \Delta_1 - (0.455 + 0.357 \Delta_1) \rho_e / \rho_w]$$

$$B_{2\infty}^{(0)} = -0.311 P_1^* - 0.194 M_3^* - \Delta_1(0.169 P_1^* + 0.134 M_3^*)$$

$$C_{2\infty}^{(0)} = 0.1 P_2^* + 0.048 M_2^* + \Delta_1 (0.0579] P_2^* + 0.0382 M_2^*)$$

$$A_{3\infty}^{(0)} = P_1^* (0.0891 \sigma_e + 0.161 \Delta_2), \quad B_{3\infty}^{(0)} = P_2^* (-0.0315 \sigma_e - 0.0721 \Delta_2)$$

$$C_{3\infty}^{(0)} = P_1^* (0.141 \sigma_e + 0.169 \Delta_2), \quad D_{3\infty}^{(0)} = P_2^* (-0.0419 \sigma_e - 0.0579 \Delta_2)$$

$$T_{3\infty}^{(0)} = \frac{1-\sigma_w}{k(1-t_0)}(1+\beta^2\varphi^2) + \sum_{i=1}^{N} \frac{c_i^e - c_i^w}{H_e - H_w}\left[\frac{1}{\sqrt{2}} h_i^w \left(1 - \frac{\sigma_w}{\mathrm{Sh}_i^w}\right) + \frac{\sqrt{2}-1}{\sqrt{2}} h_i^e \left(1 - \frac{\sigma_e}{\mathrm{Sh}_i^e}\right)\right]$$

$$F_{3i\infty}^{(0)} = \frac{\sqrt{2}-1}{\sqrt{2}} \frac{c_i^e - c_i^w}{H_e - H_w}\left[h_i^e\left(1 - \frac{\sigma_e}{\mathrm{Sh}_i^e}\right) - h_i^w\left(1 - \frac{\sigma_w}{\mathrm{Sh}_i^w}\right)\right]$$

$$A_{3+i,\infty}^{(0)} = \mathrm{Sh}_i^e (0.25\, P_1^* - 0.159\, Q_{1i}^*) + \Delta_3 (0.161\, P_1^* - 0.122\, Q_{1i}^*)$$

$$B_{3+i,\infty}^{(0)} = -(0.104 \mathrm{Sh}_i^e + 0.0721\, \Delta_3) P_2^*$$

$$C_{3+i,\infty}^{(0)} = \mathrm{Sh}_i^e (0.311\, P_1^* - 0.194\, Q_{1i}^*) + \Delta_3 (0.169\, P_1^* - 0.134\, Q_{1i}^*)$$

$$D_{3+i,\infty}^{(0)} = \mathrm{Sh}_i^e (0.048\, Q_{2i}^* - 0.1\, P_2^*) - \Delta_3 (0.0579\, P_2^* - 0.0382\, Q_{2i}^*)$$

$$\Delta_1 = 1 / l_w - 1, \quad \Delta_2 = \sigma_w / l_w, \quad \Delta_3 = \mathrm{Sh}_i^w / l_w - \mathrm{Sh}_i^e$$

After $\delta^{(0)}$, $b^{(0)}$, $d^{(0)}$ and $y_i^{(0)}$ have been determined it is possible to compute components of the friction coefficient at the wall, the enthalpy gradient and the concentration gradients

$$-l_w \frac{\partial E}{\partial \lambda}\bigg|_{\lambda=0} = \sqrt{\delta^{(0)}}\left\{N_1^*\left[0.845\left(1 - \frac{\rho_e}{\rho_w}\right) - 0.798\right] - 0.234 P_1^* + b^{(0)}(0.311 P_2^* - 0.209 N_3^*) + 0.709 N_2^* b^{(0)^2}\right\} \tag{4.4}$$

$$-l_w \frac{\partial G}{\partial \lambda}\bigg|_{\lambda=0} = \sqrt{\delta^{(0)}}\left\{M_1^*\left[0.845\left(1 - \frac{\rho_e}{\rho_w}\right) - 0.798 - 0.209 (P_1^* + M_3^*) b^{(0)} + 0.0709 (P_2^* + M_2^*) b^{(0)^2}\right\} \tag{4.5}$$

$$-\frac{l_w}{\mathrm{Sh}_i^w}\frac{\partial z_i}{\partial \lambda}\bigg|_{\lambda=0} = \sqrt{\delta^{(0)}}\{-(P_1^* - Q_{1i}^*)(0.234 + 0.209 y_i^{(0)}) + (P_2^* - Q_{2i}^*)(0.113 + 0.0709 y_i^{(0)}) b^{(0)}\} \tag{4.6}$$

$$-\frac{l_w}{\sigma_w}\frac{\partial \theta}{\partial \lambda}\bigg|_{\lambda=0} = \sqrt{\delta^{(0)}}\{-(P_1^* - R_1^*)(0.234 + 0.209 d^{(0)}) + (P_2^* - R_2^*)(0.113 + 0.0709 d^{(0)}) b^{(0)}\} + \frac{l_w}{\sigma_w}\sum_{k=1}^{4}\frac{h_k^w (c_k^e - c_k^w)}{H_e - H_w}\left(1 - \frac{\sigma_w}{\mathrm{Sh}_k^w}\right)\frac{\partial z_k}{\partial \lambda}\bigg|_{\lambda=0} \tag{4.7}$$

5. Let us consider the flow of air with frozen chemical reactions past a blunted cone with a spherical nose at an angle of attack.

As the model of air we take a four-component gas consisting of molecules, ions, atoms and electrons which we denote by M, I, A and E, respectively. Two balanced chemical reactions $M \rightleftarrows 2A$ and $I \rightleftarrows A - E$ take place at the wall. Such model makes it possible to investigate the flow of air past a body at temperatures up to 15 000° — 16 000°K downstream of the shock wave.

Computations were based on data on pressure and velocity distribution in a perfect gas on the cone surface [10].

Equations (1. 1) imply that

$$H_e = H_\infty = \text{const}, \quad c_i^e = \text{const}, \quad i = 1, \ldots, N$$

The concentrations of components of the gas are determined by conditions at the stagnation point on the assumption that the gas there is in chemical equilibrium. Then, with known H_e, U_e and c_i^e, it is possible to determine the tempersture at the outer boundary from the equation

$$\sum_{i=1}^{4} c_i^e h_i(T) = H_e - \frac{U_e^2}{2}$$

We introduce the longitudinal and transverse dimensionless components of the local friction coefficient, and the Nusselt and Reynolds numbers

$$C_{f1} = \mu_w \frac{\partial u}{\partial \zeta}\Big|_w \rho_e^{-1} u_e^{-2}, \quad C_{f2} = \mu_w \frac{\partial w}{\partial \zeta}\Big|_w \rho_e^{-1} u_e^{-2}$$

$$\mathrm{Nu} = \frac{(-I_q)\,\alpha c_p^e}{\lambda_e (H_e - H_w)}, \qquad \mathrm{Re} = \frac{u_e \gamma \rho_e}{\mu_e}$$

Taking into account the smallness of secondary flow and neglecting for a cold wall terms of order ρ_e / ρ_w, we obtain for high flight speeds $(M_\infty \geqslant 20)$ the approximate formulas

$$C_{f1} \sqrt{\mathrm{Re}} = \sqrt{\delta^{(0)}}\,(0.234\,P_1^* - 0.047\,N_1^*)$$

$$C_{f2} \sqrt{\mathrm{Re}} = C_{f1} \sqrt{\mathrm{Re}}\,\mathrm{tg}\,\gamma = \frac{w_e}{u_e} C_{f1} \sqrt{\mathrm{Re}}$$

$$\frac{1}{\sigma_e} \frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = P_1^* \sqrt{\delta^{(0)}}\,(0.234 + 0.209\, d^{(0)})$$

$$\frac{l_w}{\mathrm{Sh}_i^w} \frac{\partial z_i}{\partial \lambda}\Big|_w = P_1^* \sqrt{\delta^{(0)}}\,(0.234 + 0.209\, y_i^{(0)})$$

where γ is the angle between streamlines of the external flow and the coordinate line $\eta = \text{const}$.

The total convection flux to the wall (1. 8) is

$$I_q = -(H_e - H_w) \sqrt{\mu_e \rho_e} \sqrt{\frac{u_e}{\alpha}} \sqrt{\delta^{(0)}}\, P_1^*\,(0.234 + 0.209\, d^{(0)}) \qquad (5.1)$$

where

$$\delta^{(0)} = [0.089 P_1^* - 0.0143 N_1^* + (0.161 P_1^* - 0.031 N_1^*) / l_w]^{-1} \qquad (5.2)$$

$$y_i^{(0)} = \frac{1 - \delta^{(0)} P_1^*\,(0.089\,\mathrm{Sh}_i^e + 0.161\,\mathrm{Sh}_i^w / l_w)}{\delta^{(0)} P_1^*\,(0.142\,\mathrm{Sh}_i^e + 0.169\,\mathrm{Sh}_i^w / l_w)}$$

$$d^{(0)} = \frac{1 - (1 - \sigma_w)(1 + \beta^2 \varphi^2) / [k(1 - t_0)] - L_3 - P_1^* \delta^{(0)}\,(0.0891\sigma_e + 0.161\sigma_w / l_w)}{P_1^* \delta^{(0)}\,(0.141\sigma_e + 0.169\sigma_w / l_w)}$$

$$L_3 = \sum_{i=1}^{N} \frac{c_i^e - c_i^w}{H_e - H_w} \left[\frac{\sqrt{2} - 1}{2} h_i^e \left(1 - \frac{\sigma_e}{\mathrm{Sh}_i^e}\right)(\sqrt{2} + y_i) + h_i^w \left(1 - \frac{\sigma_w}{\mathrm{Sh}_i^w}\right)\left(\frac{1}{\sqrt{2}} - y_i^{(0)} \frac{\sqrt{2} - 1}{2}\right)\right]$$

If the coordinate system is such that the ξ-coordinate is directed along a generatrix of the cone and is measured from the leading point, and the η-coordinate, which is the angle between the meridian plane passing through the given point and the windward spreading line, the coefficients N_1^* and P_1^* are of the form

$$N_1^* = \frac{\alpha}{u_e}\frac{\partial u_e}{\partial \xi} + \frac{\alpha\beta\varphi}{\sqrt{g_{22}}\,u_e}\frac{\partial u_e}{\partial \eta} - \frac{\varphi^2\alpha\beta^2}{\sqrt{g_{22}}}\frac{\partial\sqrt{g_{22}}}{\partial \xi} \tag{5.3}$$

$$P_1^* = 0.5 + \frac{\alpha}{\sqrt{g_{22}}}\frac{\partial\sqrt{g_{22}}}{\partial \xi} + \left(\frac{1}{2}\frac{\alpha}{u_e}\frac{\partial u_e}{\partial \xi} + \frac{\alpha\beta\varphi}{2\sqrt{g_{22}}\,u_e}\frac{\partial u_e}{\partial \eta} + \frac{\alpha\beta}{\sqrt{g_{22}}}\frac{\partial \varphi}{\partial \eta} + \frac{\alpha\varphi}{\sqrt{g_{22}}}\right) + P_3 \tag{5.4}$$

$$P_3 = \frac{\alpha}{2\mu_e\rho_e}\frac{\partial \mu_e\rho_e}{\partial \xi} + \frac{\alpha\beta\varphi}{2\mu_e\rho_e\sqrt{g_{22}}}\frac{\partial \mu_e\rho_e}{\partial \eta}$$

The curves of N_1^* and P_1^* are shown in Fig. 1. The numerals 1, 2 and 3 denote in all figures the results obtained along the cone generatrices corresponding to $\eta = \pi / 20$, $\eta = \pi / 2$ and $\eta = \pi$.

Formula (5.1) and the second formula of (5.2) show that the heat flux is determined by coefficient P_1^*. The second term in (5.4) depends on the geometry of the body, the expression in parentheses depends on parameters of the external flow past the body, and the coefficient P_3 takes into account the variation of $\mu_e\rho_e$. With increasing distance

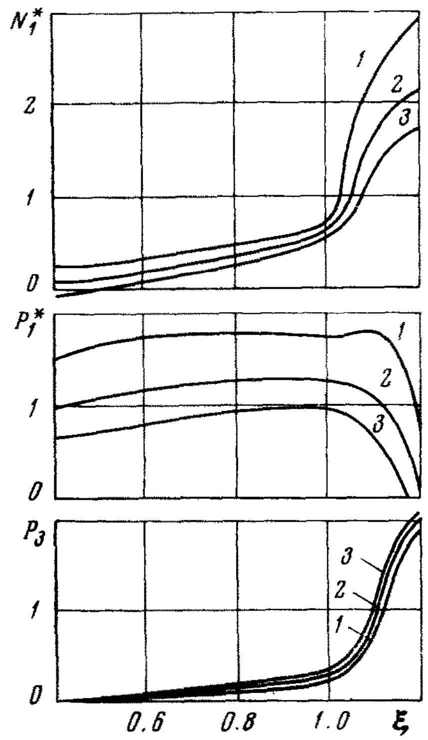

Fig. 1

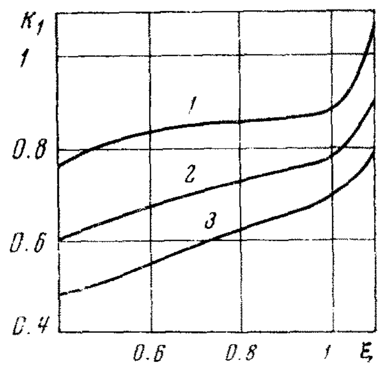

Fig. 2

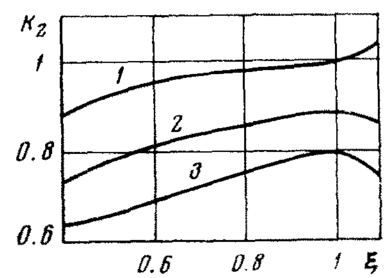

Fig. 3

from the stagnation point P_3 begins to determine P_1^* (Fig. 1), which means that the dimensionless coefficient of friction and the heat flux are then strongly affected by variation of gas properties.

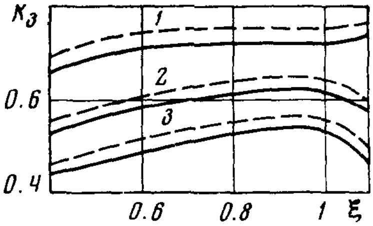

Fig. 4

It should be noted that although formula (5.1), which determines the heat flux to the wall, does not implicitly contain diffusion streams, the diffusion energy transfer in the frozen layer is taken into account in the determination of $d^{(0)}$. The heat transfer to the wall for the frozen air flow past the body is determined by the total enthalpy drop, the variation of parameter $\mu_e\rho_e$, variation of the velocity of external flow along the surface of the body, and by the geometry of the latter.

The variation of parameters $k_1 = C_{f1}\sqrt{\mathrm{Re}}$ and $k_2 = (1/\sigma_e)\,\mathrm{Nu}/\sqrt{\mathrm{Re}}$ along the cone generatrices is shown in Figs. 2 and 3, while the gradients of dimensionless concentrations $k_i = (l_w / \mathrm{Sh}_i^w)(\partial z_i / \partial\lambda)_w$ of molecules and ions (dash lines) appear in Fig. 4.

The effect of the considered model is apparent on the magnitude of numerical coefficients in (5.1) — (5.3) and on values of controlling coefficients $\delta^{(0)}$, $d^{(0)}$ and $y_i^{(0)}$.

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